A Contribution to the Theoretical Prediction of Life-time in Glass Structures

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Summary

In order to assess safety levels in glass structures a scattered and inhomogeneous variety of mostly complicated resistance criteria is presently available, very often requiring specially developed softwares. For this reason engineers who wants to assess with reliability the actual safety level of glass structures of relevant economical importance are still obliged to undertake expensive experimental tests.

In the attempt to overcome this problem, it was formulated a new semi-probabilistic failure prediction method called "*Design Crack Method*" (DCM), which is a compromise between the necessity to accurately model the complex mechanical behaviour of glass at breakage and the need to reduce the analytic complexity of the calculations. On the basis of Linear Elastic Fracture Mechanics, such aim has been reached in the present work by defining a new quantity called *Design Crack*, characterized by a mathematical expression that depends only on the probability of failure and on the surface damaging level.

The proposed method, which is in accordance with the basic principles of the Structural Eurocodes, allows to predict glass lifetime taking into due account the influence of parameters like the surface extension and the loading time-history of the structural element. The results obtained in some applications with the D.C.M. method have been numerically compared in this paper with those of the existing most frequently used theoretical methods.

Keywords:

glass strength, design crack, fracture mechanics, static fatigue, surface flaws, life time, probabilistic model, analytical method.

1 Introduction

Glass is increasingly used in modern Architecture to resolve non secondary structural tasks.
As a result there has risen an urgent demand for a reliable, theoretical method to perform engineering assessments of the remaining lifetime in glass structure.
The availability of such a method would permit a safe design of glass structures and the prediction of their mechanical behaviour even if, like presently, the data base of experimental results obtained on full scale structures is not yet sufficiently extensive.
The peculiar sensitivity of glass to surface imperfections, the absence of plasticity and phenomena like the so called "static fatigue" for example and the diminished resistance of the material to long term loads prevents the use of the traditional calculation methods that apply for other building materials.
Also standard safety assessment procedures specifically developed for windows cannot be employed for glass beams or glass floors because window glasses must prevalently face short time loadings such as wind gusts.
The first scientifically consistent, but engineering oriented, theoretical model of glass strength was formulated in 1972 by Brown [1] who called it "Load Duration Theory" (LDT). Brown combined the static fatigue theory of Charles & Hillings [2] with the concept of failure probability expressed by Weibull [3].
In 1974 Evans [4] developed the "Crack Growth Model" (CGM) on the basis of the principles of Linear Fracture Mechanics. This method makes use of the empirical description of the sub-critical propagation of cracks (deduced from the experimental relationship between crack growth speed and stress intensity factor K_I) together with the Weibull failure probability concept under the hypotheses that in all surface micro cracks a sub-critical crack growth takes place no matter how little their stress intensity factor K_I is.
Between 1980 and 1984 Beason and Morgan formulated the "Glass Failure Prediction Model" (GFPM) [5] devoted to the safety assessment of rectangular glass plates simply supported along their borders.
Here also the static fatigue theory of Charles & Hillings is used together with the Weibull concept of failure probability. Calculated stresses include geometrical non linearities caused in thin plates by large deflections.
The GFPM constitutes the theoretical fundament of Canadian Standard CAN/CGSB 12.20-M89 and of ASTM 1300-04.
Nevertheless, Fisher-Cripps & Collins [6] demonstrated in 1994 that the GFPM was not able to predict glass cracks under short time not under long time loads. However, the CGM also demonstrated a lack of capability of predicting fractures under long time loads. In order to obviate this problem, the same Authors introduced in CGM an additional, experimentally based condition which states that any crack growth occurs if the K_I factor is less than a limit value called Static Fatigue Limit. The new approach was called Modified Crack Growth Model (MCGM) [6].
Between 1995 and 1999 Sedlacek and Others [7] developed an engineering method to check structural safety in glass structures based on Weibull failure probability and on Linear Fracture

Mechanics. The final verification expression obtained by Sedlacek is formally similar to the Miner rule of progressive damaging, commonly used by designers to assess the remaining fatigue life of steel structures.
In 2001 Shen [8] and Siebert [8] proposed their own versions of Sedlacek method, both not very different from the original formulation and giving comparable results.
The Sedlacek approach is presently at the basis of Eurocode prEN 13474-3.
In the same year Porter [10] elaborates the Crack Size Design (CSD) where he defines a "design crack", i.e. a maximum design depth that surface cracks, supposed uniformly distributed on a glass pane, may reach before failure.

In 2006 Haldimann gave an important contribution to the solution of this problem with his Lifetime Prediction Model (LPM) where he avoids the introduction of equivalent quantities but calculates directly the failure probability of a glass element starting from the probability distribution of its defects and from the deterministic knowledge of loading time-history [11] .
Devigili in the same year [12] removed also the conceptual limitation, in Haldimann's theory, of the deterministic definition of the time history by introducing the hypothesis that the random properties of the surface micro-defects and of the loading time histories can be described by Markov's probability distributions.
The results appear very rigorous but the associated calculation difficulties prevent the application of Devigili's method for current engineering purposes.
In this paper we tried to formulate a method for extending LPM but maintaining at the same time a moderate level of analytical difficulty in order to let its application remain possible also for normal design activities.

2 Basic Concepts about the Mechanical Behaviour of Glass

Every glass surface, although apparently intact, is inevitably affected by microscopic randomly distributed cracks. When the glass element is subjected to mechanical stresses, high stress concentrations occur at the tip of the micro cracks which can not be plastically redistributed because of the amorphous crystalline structure of the material, lacking in preferential plastic-flow plans. This peculiar feature causes the typical brittle fractures that characterize this material.

The fracture resistance of damaged elements can be analytic described by the principles of *Linear Elastic Fracture Mechanics*. For this reason Irwin [13] introduced the *Stress Intensity Factor (K)*, in order to describe the behaviour of brittle materials damaged by a single flaw placed perpendicularly to the stress direction (opening mode I):

$$K(t) = \sigma(t) Y \sqrt{\pi \cdot a(t)} \tag{2-1}$$

Where :

- Y - Shape factor, that depends on flaw's geometry and dimension;

- $\sigma(t)$ - Time-history of the tensile stress near to the crack edge;
- $a(t)$ - Time-history of crack depth.

Failure occurs when the propagation of the crack becomes unstable; that happens when:

$$K(t) \geq K_{IC} \tag{2-2}$$

K_{IC} represents the *Critical Stress Intensity Factor* which depends only on the kind of material and can be usually considered technically constant because of its low statistical spread. Substituting (2-2) in (2-1) easily allows to obtain the $\boldsymbol{a_{cr}}$ e $\boldsymbol{\sigma_{cr}}$ analytic expressions, respectively representing the crack depth and the stress intensity able to induce unstable crack propagation. This pair of values identifies the so-called "*inert strength*".

The graph of Fig. 2-1 shows, according to the K-factor, the flaw propagation velocity of a glass element subjected to constant stress during time and immersed in a humid environment.

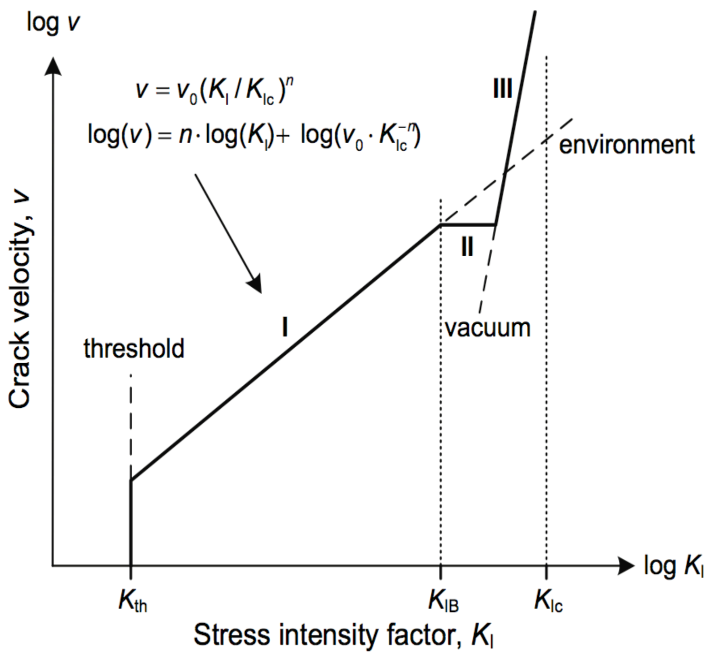

Fig. 2-1 – υ versus K_I

Although in section I the K value is much lower than K_{IC}, a slow sub-critical growth of flaws depth occurs on the glass surface which gradually reduces the inert tensile glass strength over time. This phenomenon is known as *static fatigue* and plays one of the main roles in theoretically determining the ultimate strength of glass structures.

Moreover, the only part of Fig. 2-1 that provides a significant contribution to the design life of a crack, which is submitted to stress intensification during time, is the section I because the failure of the glass plate occurs almost instantly when section III is reached. As shown in Fig.2-1, the *v*-K relation is represented by a constant slope curve on a bi-logarithmic plot and it can be analytically described by the following differential equation:

$$\frac{\partial a}{\partial t} = v_0 \left(\frac{K(t)}{K_{IC}} \right)^n \tag{2-3}$$

n being the curve's slope in section I and v_0 the propagation velocity when $K = K_{IC}$. Both *n* and v_0 are generally dependent on relative humidity rate, temperature, pH and stress intensity.

It can be easily argued that the theoretical failure prediction of glass elements subjected to tensile stress histories and afflicted by unavoidable surface flaws is rather complex and highly dependent on the following factors:

- Environment conditions;
- Surface damaging rate;
- Tensile stress history on the glass surface during time;
- Element shape and restrains;
- Presence of residual stress induced by tempering processes.

3 Haldimann's Probabilistic Method: *the Lifetime Prediction Model*

At present, the most advanced prediction model of strength in glass element seems to be the *Lifetime Prediction Model* formulated by M.Haldimann in which he assumes the following hypothesis:

- The crack depth is a random variable;
- All glass elements contain a large number of flaws;
- The life of the entire element coincides with that of the single worst defect ;
- Cracks do not influence each other;
- Generic flaw positions and orientations have the same probability of occurrence;

Haldimann also demonstrates that the crack opening mode I mainly affects failure probability (P_f) and at last he finally finds the following general expression of P_f that describes the life-time of a glass element of whatever shape, submitted to sub-critical cracks' growth and to a generic time and space variable stress-history :

$$P_f(t) = 1 - \exp\left\{ -\frac{2}{A_0 \pi} \int_A \int_{\varphi=0}^{\pi/2} \left[\max_{\tau \in [0,t]} \left\{ \left(\left(\frac{\sigma(\tau, \vec{r}, \varphi)}{\vartheta_0} \right)^{n-2} + \frac{1}{U \vartheta_0^{n-2}} \int_0^{\tau} \sigma^n (\tilde{\tau}, \vec{r}, \varphi) d\tilde{\tau} \right)^{\frac{1}{n-2}} \right\} \right]^{m_0} dA d\varphi \right\} \tag{3-1}$$

Where:

- A is the surface area of the glass element ;
- θ_0 and m_0 are statistical parameters, related to the Weibull distribution, that describe the damage rate of the surface. They can be determined by experiments and are material intrinsic properties, not dependant on the type of laboratory tests.
- A_0 is the reference surface of the element used to obtain θ_0 and m_0 ;

- $U = \dfrac{2K_{IC}^2}{\left[(n-2)v_0 Y^2 \pi\right]}$ is an expression related to specific parameters of the material, usually characterized by constant values, defined by *Linear Elastic Fracture Mechanics* and by the *Static Fatigue* differential equation.

The (3-1) is therefore related, by means of a probabilistic approach, to the parameters θ_0 and m_0 which are characterized by a clear physical meaning. Restrictive assumptions are not stated about element shape, load or stress time-history and space variability, constraints and damaging surface condition. The only conceptual limitation of the Lifetime Prediction Model is that loads are assumed to be deterministic variables.

The analytical complexity expression (3-1) make it not suited for current engineering oriented design activities. For this reason, Haldimann himself suggested a simplified version of it by introducing the following simplifying but conservative assumptions:

- The existence of the threshold stress intensity factor K_{th} can be ignored;
- It is assumed that the surface stress field is characterized by equi-biaxial tensile stresses $\sigma_1 = \sigma_2$;
- The Brown's Integral is assumed to apply (see paragraph 4).

After some manipulation, expression (3-1) finally reduced to the following simplified expression of the failure probability (the meaning of $\overline{\sigma}$ is described in paragraph 5 by expressions (5-1) and (5-2):

$$P_f = 1 - \exp(-k \cdot \overline{\sigma}^{m'}) \tag{3-2}$$

Where:

$$k = \frac{t_0}{U \cdot \vartheta_0^{\,n-2}} \tag{3-3}$$

$$m' = \frac{n \cdot m_0}{n-2} \tag{3-4}$$

Then, once selected a given P_f for the glass tensile strength, a failure criterion can be written in the following form:

$$\text{Stress} \;=\; \boxed{\overline{\sigma} \le \left[-\ln(1-P_f)\right]^{1/m'} \cdot \left(\frac{t_0}{U \cdot \theta_0^{n-2}}\right)^{\frac{-1}{n}}} \;=\; \text{Strength}_{\,probabilistic} \tag{3-5}$$

Haldimann's Lifetime Prediction Model has been adopted as the theoretical starting point for the development of the new analytic model proposed in this paper, which has been called *Design Crack Method* and is described in the next paragraph 5.

4 Deterministic Model: *Single Crack Life Time*

The (2-1) and (2-3) describe glass mechanical behaviour during time of an ideal perfect element only damaged with a single flaw, referring respectively to the Linear Elastic Fracture Mechanics and to the static fatigue phenomenon. Let us suppose that the general surface stress time-history is uniform over the surface and acting for T-seconds. If the initial crack size is also known, substituting (2-1) in (2-3) yields to the following integral-differential equation with separable variable:

$$\int_0^t \sigma^n(\tau)\left[v_0\left(\frac{Y\sqrt{\pi}}{K_{IC}}\right)^n\right]d\tau = \int_{a_i}^{a_f} a^{-\frac{n}{2}}\,da \tag{4-1}$$

From (4-1), assumed in accordance with Haldimann [11] that n e v_0 are constant with time, integrating between time $t = 0$, when the crack depth is a_i , and a generic time t , when the crack depth is a_f , we obtain the expression (4-2) describing the crack evolution at the generic instant t as a function of the load-history:

$$a_f = a(t) = \left[a_i^{\frac{2-n}{2}} + \frac{2-n}{2\cdot K_{IC}^n} v_0\left(Y\sqrt{\pi}\right)^n \cdot \int_0^t \sigma^n(\tau)d\tau\right]^{\frac{2}{2-n}} \tag{4-2}$$

Formula (4-2) can be expressed also in the following way:

$$\int_0^t \sigma^n(\tau)d\tau = \frac{2\cdot K_{IC}^n}{(n-2)v_0(Y\sqrt{\pi})^n a_i^{\frac{n-2}{2}}}\left[1-\left(\frac{a_i}{a(t)}\right)^{\frac{n-2}{2}}\right] \tag{4-3}$$

Taking into account of (2-2), formula (4-3) can be rearranged into (4-4) which states that a brittle unstable crack propagation does not occur if the following condition is satisfied:

$$\int_0^T \sigma^n(t)dt \le \frac{2\cdot K_{IC}^n}{(n-2)v_0(Y\sqrt{\pi})^n a_i^{\frac{n-2}{2}}}\left[1-\left(\frac{a_i}{\left(\frac{K_{IC}}{\sigma(t)Y\sqrt{\pi}}\right)^2}\right)^{\frac{n-2}{2}}\right] \tag{4-4}$$

Finally, assuming the conservatory hypothesis that $a_i <<< a_{cr}$ (as demonstrated by Haldimann [11] for common load application durations), the quantity in square brackets approaches 1 and therefore it is possible to separate the variables obtaining the following inequality where the damage caused by external loading is compared with the maximum damage that glass can withstand:

$$D_{amage_{Soll.}}(\sigma,t) \le D_{amage_{Max.Tolerable}}(K_{IC},n,v_0,Y,a_i) \tag{4-5}$$

If we rewrite (4-5) in an explicit way we obtain:

$$\int_0^T \sigma^n(t)dt \le \frac{2K_{IC}^n}{(n-2)v_0(Y\sqrt{\pi})^n} a_i^{\frac{2-n}{2}} \tag{4-6}$$

where the first part of (4-6) is the well known *Brown's Integral*.
Therefore, if we know a generic stress time-history lasting T-seconds, by arbitrarily choosing the value of t_0 reference time, it is possible to calculate by Brown's Integral the equivalent constant tensile stress that induces onto the glass surface, during t_0 , the same damage as the real stress history variable over the time T:

$$\sigma_{t_0} = \left[\frac{1}{t_0}\int_0^T \sigma^n(t)dt\right]^{\frac{1}{n}} \tag{4-7}$$

With position (4-7) expression (4-6) takes the form:

$$\text{Stress} \quad = \quad \boxed{\sigma_{t_0} \le \left[\frac{1}{t_0}\frac{2K_{IC}^n}{(n-2)\cdot v_0\cdot (Y\sqrt{\pi})^n} a_i^{\frac{2-n}{2}}\right]^{\frac{1}{n}}} \quad = \quad \text{Strength}_{\ deterministic} \tag{4-8}$$

5 The Design Crack Method (DCM)

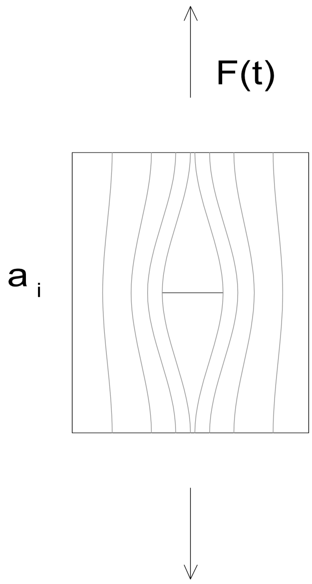

Fig. 5-1 – Basic Idea [14]

5.1 Basic idea

In paragraph 4 we have briefly recalled the deterministic model of the mechanical behaviour of an ideal perfect glass plate, only containing a single flaw with a known initial depth a_i, submitted to a uniform tensile stress $\sigma(t)$ generically variable during a time T.

The two main advantages of this model consist of its analytical simplicity and of the reliability of the solution, but on the other hand it does not take into account the randomness of the main parameters affecting the problem of glass tensile strength during time.

In order to overcome this problem, it was thought to search the analytical expression of a single *“**Design Crack**”* having such a depth $\boldsymbol{a_{i,d}}$ able to induce the same damaging rate of the real glass element subject to a random distribution of cracks over its surface. It was also decided to pursue this goal analytically, without using any empirical coefficient or assumption.

In the following, $\sigma_{t0}(x,y)$ represents the constant tensile stress of duration t_0 equivalent to the real stress history $\sigma(t,x,y)$ generally variable during time T, while $\overline{\sigma}$ represents the uniformly distributed and constant over time t_0 tensile stress acting across the element of area A, equivalent to (in terms of damage) any generic $\sigma(t,x,y)$ (see also [14]).

Analytically :

$$\sigma_{t_0}(x,y) = \left[\frac{1}{t_0}\int_0^T \sigma^n(t,x,y)dt\right]^{\frac{1}{n}} \approx \left[\frac{1}{t_0}\sum_{i=1}^{k}\sigma(x,y)_i^n \cdot \Delta t_i\right]^{\frac{1}{n}} \tag{5-1}$$

$$\overline{\sigma} = \left[\frac{1}{A_0}\int_A \sigma_{t_0}^{m'}(x,y)dA\right]^{\frac{1}{m'}} \approx \left[\frac{1}{A_0}\sum_{j=1}^{q}\sigma_{t_0,j}^{m'} \cdot \Delta A_j\right] \tag{5-2}$$

In (5-1) and (5-2), k is the number of approximately constant time-intervals and q is the number of surface regions subject to approximately uniform surface tension, where obviously the integrals are extended only to decompressed areas, referring to surface stress field net of compression residual stresses induced by tempering processes and external pre-stressing.

5.2 Analytical Formulation of the Design Crack Method

The problem is analytically stated by equating the material strength expressed in a deterministic way by the second term of equation (3-5) to the probabilistic strength expressed by equation (4-8):

$$R_{\det}(v_0,n,Y,K_{IC},t_0,a_{i,d}) \quad = \quad R_{probabilistica}(P_f,\vartheta_0,m_0,n,v_0,Y,K_{IC}) \tag{5-3}$$

Realising all terms of (5-3) we obtain:

$$\left[\frac{1}{t_0}\cdot\frac{2\cdot K_{IC}^n}{(n-2)\cdot v_0\cdot\left(Y\sqrt{\pi}\right)^n}\cdot a_{i,d}^{\left(\frac{2-n}{2}\right)}\right]^{1/n} = \left[-\ln\left(1-P_f\right)\right]^{1/m'}\cdot\left(\frac{1}{t_0}\cdot\frac{2\cdot K_{IC}^2}{\left[(n-2)\cdot v_0 Y^2\pi\right]}\cdot\vartheta_0^{(n-2)}\right)^{1/n} \tag{5-4}$$

It can be seen that the time-term t_0 can be eliminated and therefore, after some re-arranging, we achieve the final time-independent expression of the ***Design Crack*** $a_{i,d}$,

$$\boxed{a_{i,d}=\left[\left(\frac{K_{IC}}{Y\vartheta_0}\right)^2\frac{(-\ln(1-P_f))^{-2/m_0}}{\pi}\right]} \quad = a_{i,d}\,(P_f,\ \theta_0,\ m_0) \tag{5-5}$$

Combining (5-5) with (4-5), we define at first the failure criterion in terms of damage:

$$D_{anno_{Soll}}(t,\sigma) \quad \leq \quad D_{anno_{Max.Accumulabile}}(a_{i,d}) \tag{5-6}$$

After some re-arrangement, the preceding failure criterion related to a glass element submitted to any stress time-history over a time interval can be written in more common terms of tensile stresses:

$$\text{Stress} \;=\; \boxed{\bar{\sigma}\leq\left[\frac{1}{t_0}\frac{2K_{IC}^n}{(n-2)\cdot v_0\cdot(Y\sqrt{\pi})^n}\,a_{i,d}(P_f,\vartheta_0,m_0)^{\frac{2-n}{2}}\right]^{\frac{1}{n}}} = \text{Strength}_{\,Semi\text{-}Probabilistic} \tag{5-7}$$

The final glass strength equation illustrated by equation (5-7) is thus reached by using the probabilistic parameters $a_{i,d}$ (P_f , θ_0 , m_0) defined by (5-5) together with the deterministic strength criterion described by (4-8).

It can be affirmed that the present criterion belongs to the so-called semi-probabilistic safety verification processes (level 1), where t_0 is arbitrarily chosen and K_{IC}, n, Y , thanks to their low statistical spread [15], can be technically characterized by constant values. As demonstrated in [14] the variations of other parameters do not influence significantly the solution of equation (5-7) which exhibits a stable behaviour.

In Design Crack Method, the failure probability is directly contained in (5-5) and the aspects linked to glass material characteristics comes into play by the parameters θ_0 e m_0 obtained by Haldimann in [11], statistically analyzing a large number of failure tests by L.P.M changing some main factors such as geometry, environmental condition, load shape and load increasing velocity. For this reason Haldiman's values of θ_0 e m_0 are characterized by the highest reliability level [14] and they will be adopted for the following numerical applications.

The following scheme summarizes the developing process of Design Crack Method, based on the comparison between the simplified probablilistic Haldiman's method and the deterministic failure prediction of a single crack analyzed by Linear Elastic Fracture Mechanic:

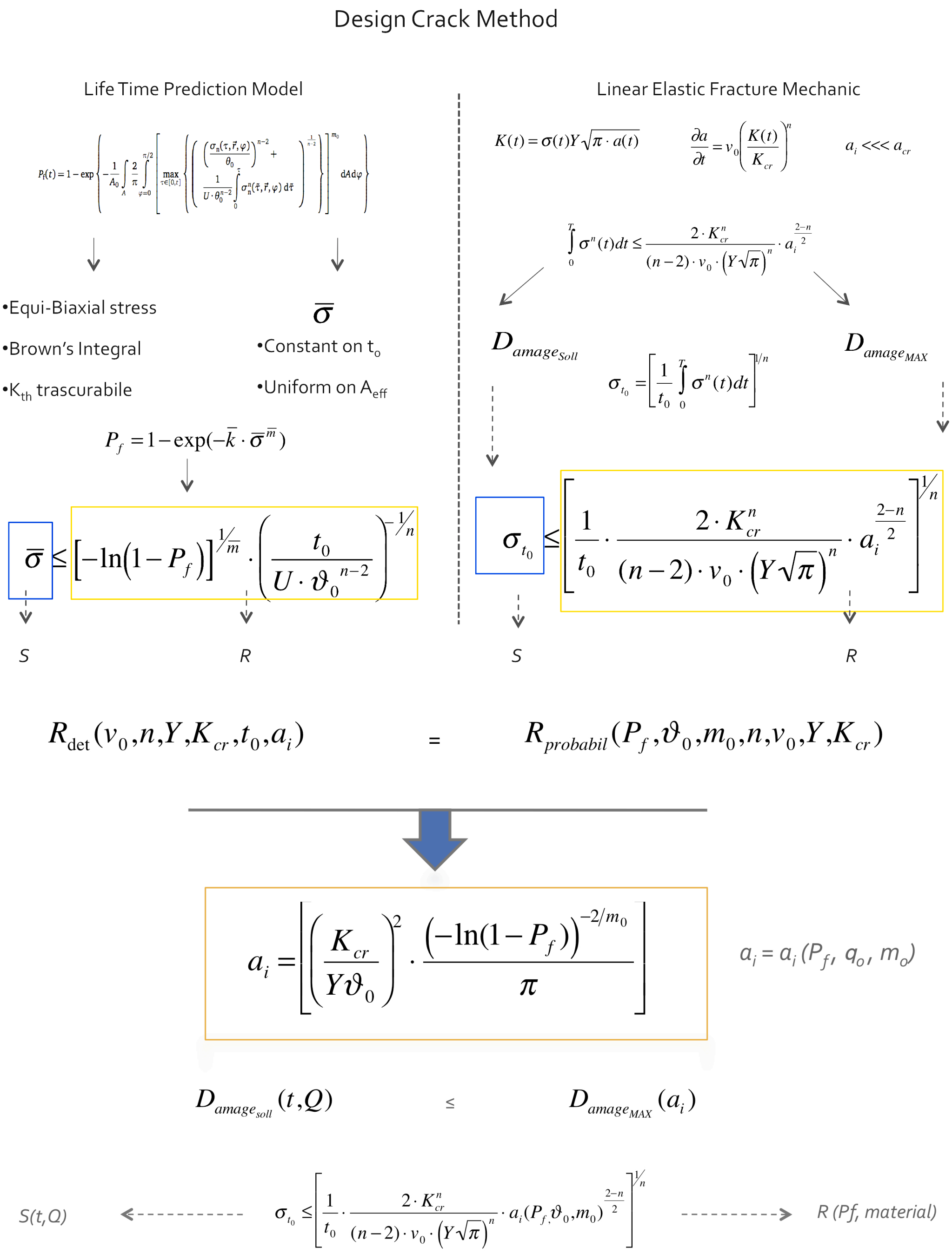

Fig. 5.2 – Design Crack Method Developing Process [14]

6 Numerical Applications

6.1 Existing Methods

6.1.1 Comparisons

This paragraph presents the results of a numerical comparison performed today on the most commonly used criteria of glass strength [14]. The comparison is not immediate because some authors choose the failure probability as the most relevant quantity for the prediction of the safety level, others use the maximum uniform out-plan load, or the allowable stress, and finally other authors transform the service condition to an equivalent standard laboratory test.

In order to make the comparisons possible it was necessary to restate and homogenise to a unique approach, all the different calculation methods. The rearranging was made in [14] following two ways: the first way was to deduce from each method the failure probability when applying the same constant uniform tensile stress. Inversely, the second way consisted in calculating for each method, given a constant failure probability, the associated maximum constant tensile stress.

The glass element object of the present numerical application is a square plate 1m x 1m, 4mm thick, uniformly loaded, simply supported at the edges (free edges rotations), subjected to two different stress time-histories: short duration (60s) and long duration (50 years). The choice of two very different loading times was necessary since glass is very sensitive to the so-called *Static Fatigue* effect. The reference values of tensile stress and failure probability used in the comparison are compiled in Table [14].

Table 6-1 - Reference values [14]

	Short Time (60s)	*Long Time (50 years)*
σ_{max}	20.3 MPa	8.12MPa
P_f	$8x10^{-3}$	$8x10^{-3}$

Instead of comparing single result values, it was decided that it would be more significant to make a comparison between the different distributions of the failure probability as a function of the applied uniform tensile stress for each of the two loading times (see Figs. 6-1; 6-2).
In the following tables bolded values resulted in accordance with reference values .

6.1.1.1 P_f - Short time duration

	Model	P_f
1	**L.D.T.**	**6.092x10^{-3}**
2	**C.G.M.**	**5.621 x10^{-3}**
3	*G.F.P.M.*	*2.344 x10^{-4}*
4	**M.C.G.M.**	**5.621x10^{-3}**
5	**Sed.**	**3.091 x10^{-3}**

Input	*Short time*
σ_{to}	20.3 Mpa
t_0	60 s
UR	100%
T	25 °C
S_v	5
n_v	16

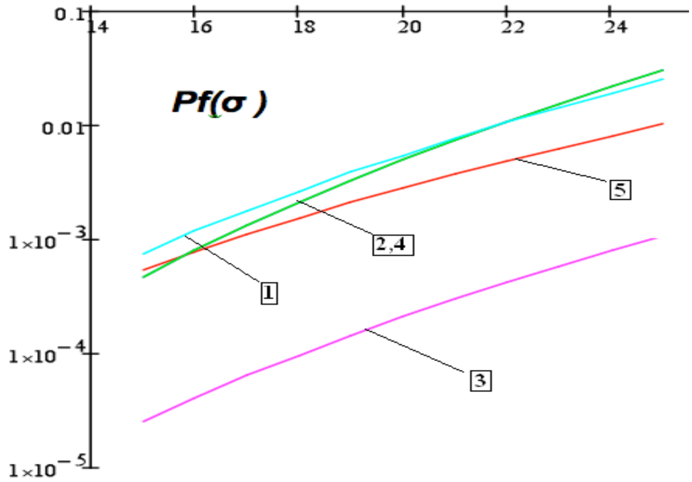

Fig. 6-1 - P_f function vs constant stress – Short Duration – [14]

6.1.1.2 P_f - Long time duration

Input	*Long time*
σ_{to}	8.12 MPa
t_0	50 years
UR	50%
T	25 °C
S_v	0.45
n_v	18.1

	Model	P_f
1	*L.D.T.*	*0.015*
2	*C.G.M.*	*0.089*
3	*G.F.P.M.*	*1.060 x10^{-3}*
4	**M.C.G.M.**	**6.753 x10^{-3}**
5	**Sed.**	**7.724 x10^{-3}**

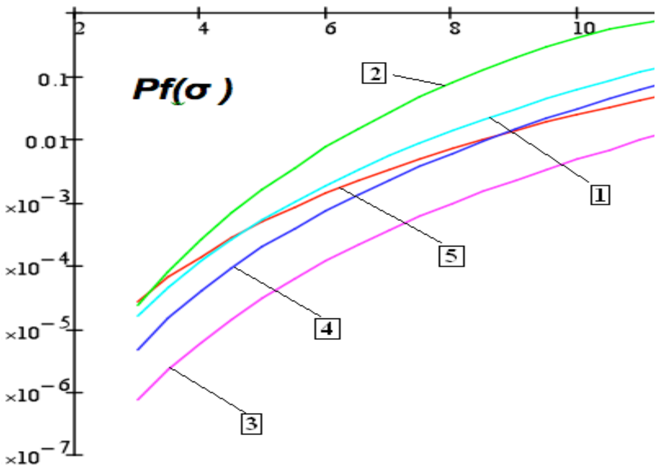

Fig. 6-2 - P_f function vs constant stress– Long Duration – [14]

6.1.1.3 Stress – Short time duration

Input	*Short Time*
P_f	8x10^{-3}
t_0	60 s
UR	100%
T	25 °C
S_v	5
n_v	16

	Model	σ_{to} [MPa]
1	**L.D.T.**	**21.119**
2	**C.G.M.**	**19.613**
3	*G.F.P.M.*	*32.942*
4	**M.C.G.M.**	**21.174**
5	**Sed.**	**20.664**

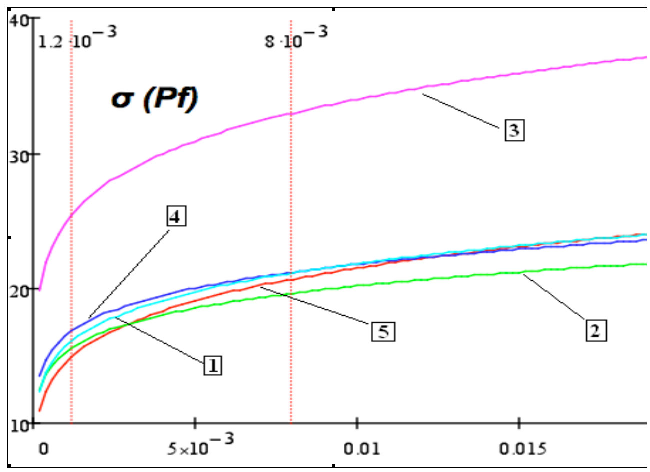

Fig. 6-3 – Constant Stress vs P_f – Short Duration [14]

6.1.1.4 Stress – Long time duration

Input	*Long Time*
P_f	$8x10^{-3}$
t_0	50 years
UR	50%
T	25 °C
S_v	0.45
n_v	18.1

	Model	σ_{to}
1	*L.D.T.*	*7,413*
2	*C.G.M.*	*4,769*
3	*G.F.P.M.*	*10,715*
4	**M.C.G.M.**	**8,311**
5	**Sed.**	**8,198**

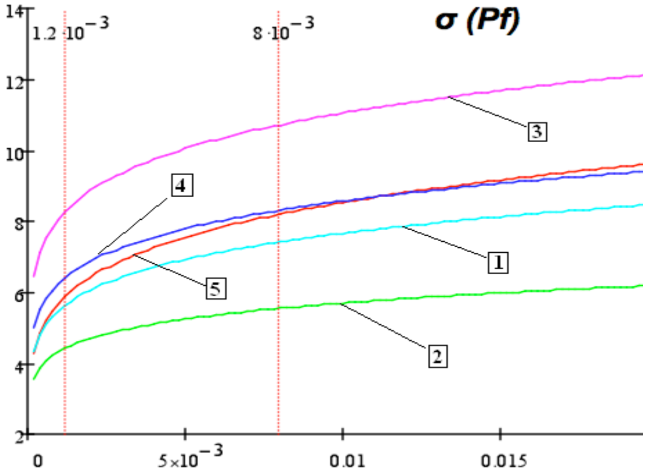

Fig. 6-4 - Constant Stress vs P_f – Long duration – [14]

6.1.2 Comments

The following main remarks can be deduced from the numerical results:

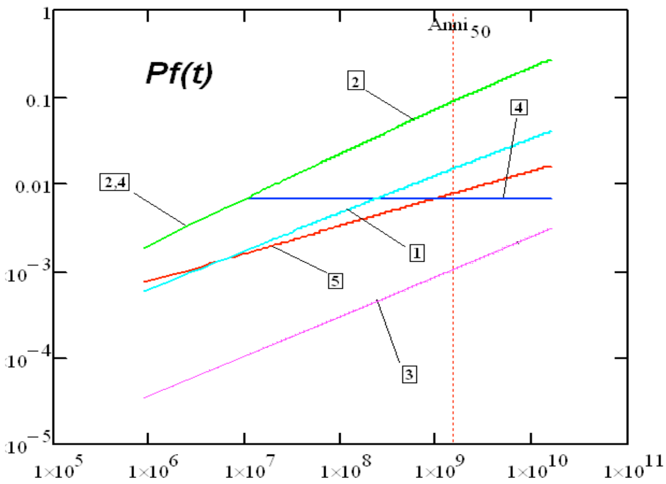

Fig. 6-5 [14] – Trand of P_f over time – constant stress

- Brown affirms that his L.D.T calculation method can be applied only for short duration loads, which is confirmed by the graphs of the preceding figures. Indeed that was the reason why the C.G.M. was modified into the M.C.G.M., which shows a trend of P_f that can be described by a bilateral straight line in a bi-logarithmic graph, as shown in Fig. 6-5. After a certain time, depending on the stress intensity, the C.G.M's curve is interrupted and replaced with a new almost zero-slope line along which the failure probability does not increase. Thanks to this modification, the long duration solution also belongs to intervals of P_f with a magnitude of 10^{-3} instead of 10^{-1}.

- Our numerical results show also that the G.F.P.M does not provide safety values for either long or short time loading. In spite of that, this method is still adopted by some national standards like the American ASTM E 1300 and the Canadian CAN 12-20 since its application leads to results in accordance to reference values. However, it can be demonstrated [15] that this is surprisingly and solely due to compensating errors contained in some formulas.

The Sedlacek's calculation model gives stress and P_f values similar to the reference ones, both for long and short loading time-histories. The assessment process is performed by transforming the real service condition of a generic glass element into an equivalent standard laboratory test, the so-called *double ring test [15, 16],* by means of a set of coefficients whose knowledge and reliability is implicitly assumed [7].

- In some of the examined existing methods, it was not clearly defined which was the extension of the surface to be introduced in the different formulas. In our opinion, since just tensile stressed opening cracks increase the failure probability, only the decompressed surfaces should be taken into account.

6.2 Design Crack Method

Under the same service conditions of restrains, geometry and environment used in paragraph 6-1, some simple numerical applications of the new D.C.M. have been carried out with the results summarized in Table 6-2.

Also developed was a numerical simulation of the standard double-ring test in application n°3 while in application n°4 the numerical convergence of the method to the inert strength a_{cr} e σ_{cr} was controlled, for a extremely high load application rate, by numerically solving expression (4-4) instead of (4-6) .

A very good agreement between predicted and reference values can be observed both for short and long time loading applications.

Table 6-2 : D.C.M. - Numerical Applications – [14]

n°	**Numerical Applications**	**Design Crack Method**	**Reference Values**
1.1	σ_{max} – **60 s**	21.76 *MPa*	20.3 *MPa*
1.2	σ_{max} – **50 anni**	8.27 *MPa*	8.12 *MPa*
2.1	P_f - ***60 s***	4.122 x 10^{-3}	8 x 10^{-3}
2.2	P_f - ***50 anni***	6.734 x 10^{-3}	8 x 10^{-3}
3	$\sigma_{bB,A0}$	42.24 *MPa*	45 *MPa*
4	σ_{max} – $10^{-10} \approx 0$ s	34.68 *MPa*	σ_{inerth}=34.68 *MPa*

7 Conclusion

The present panorama of glass strength criteria that can be applied in the field of architectural glass structures is still far from being simple and homogeneous. Despite the wide variety of existing calculation models, none of them are preferred by glass designers or researchers since some methods are usable only for simple geometries and standard constraint set ups, while others can be applied to any element and service condition but are difficult to use due to their high calculation complexity. On the other hand, some other methods demonstrated to be totally inadequate because of their lack of sufficiently safe results.
With the new failure prediction method proposed in this paper we tried to develop a user-friendly tool while maintaining at the same time the precision of the most rigorous methods but avoiding their high calculation burden. This has been done with the aim of supporting structural engineers when tasked with designing glass structures.
This aim has been pursued through the adoption of a reliable, easy to understand deterministic model, already used by designers, together with the simplified version of the more accurate glass strength model developed by Haldimann.

The new method, called *Design Crack Method*, is free from any empirical formulation. The crucial assessment parameter is a crack's depth, called *Design Crack*, which is independant on time and takes into account all the main factors that statistically govern the strength of glass during time, such as the static fatigue and the stress concentration at the apex of the surface flaws.

The great sensitivity of glass strength on the sequence and duration of external loading highlights the importance of defining significant stress time-history as standard loading conditions, a problem that could be solved within the frame of Eurocode activities.

8 Appendice A: Numerical Stability of the Solution

Some tests have been performed to control the numerical stability of the Design Crack Method. In these tests the geometry of the glass plate and other service conditions have been taken similar to those in paragraph 6 for 60s load duration.
As already described in Chapters 2, the values of K_{IC}, n and Y can be assumed technical constants [15] during design and verification processes. The following graphs show therefore the trend of the Design Crack Method's solution as a function of the remaining parameters.

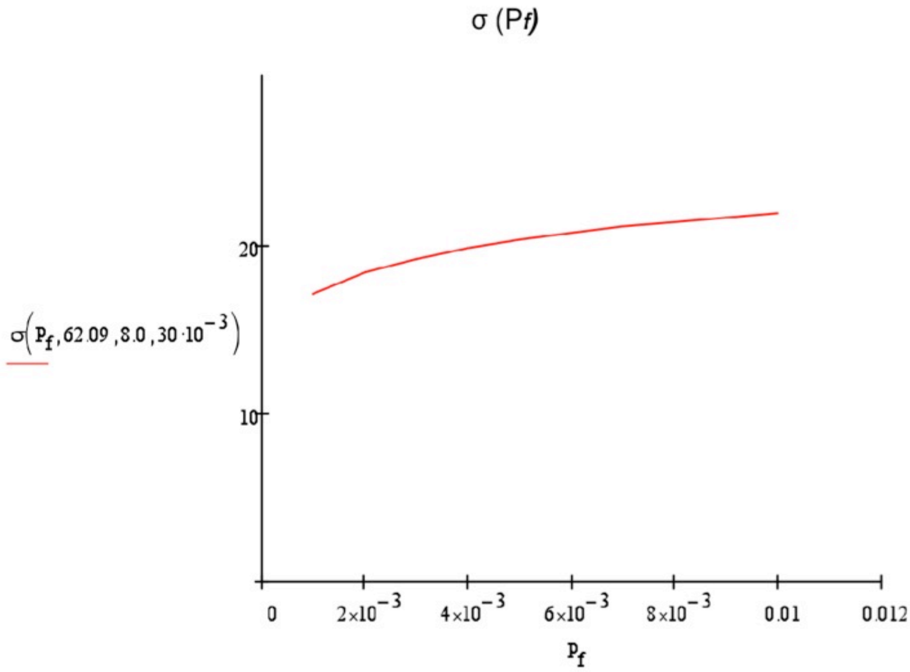

Fig. 8.1.a – Failure uniform tensile stress vs P_f [14]

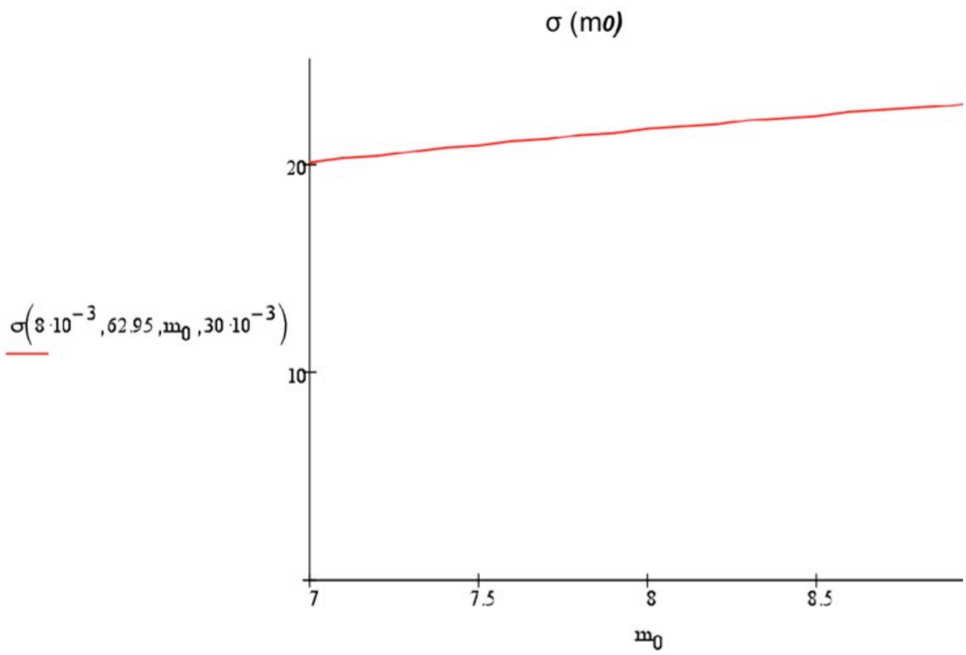

Fig. 8.1.b – Failure uniform tensile stress vs m_0 [14]

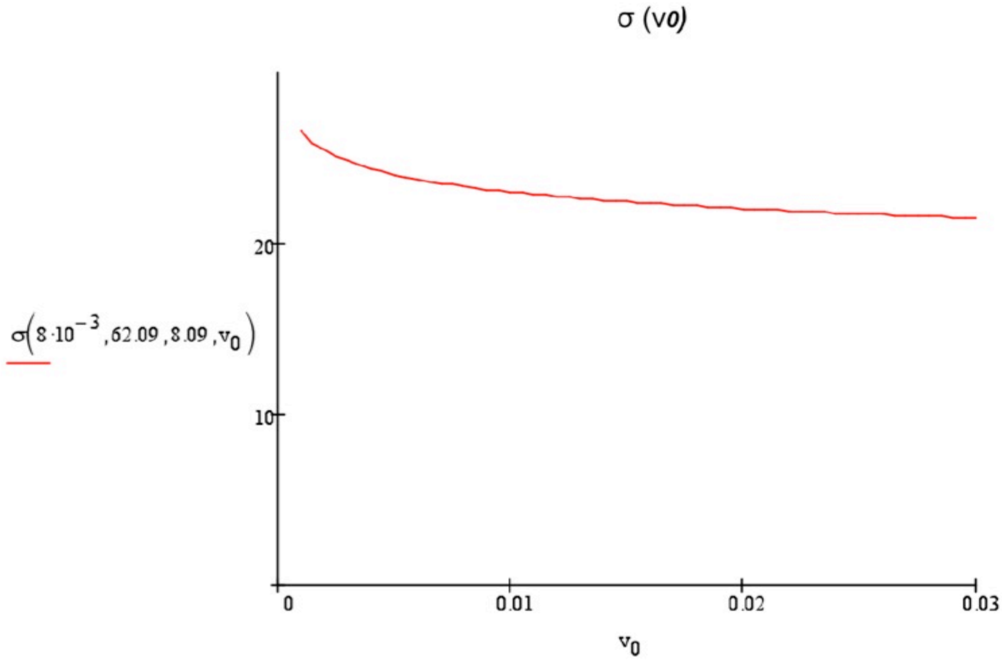

Fig. 8.1.c – Failure uniform tensile stress vs v_0 [14]

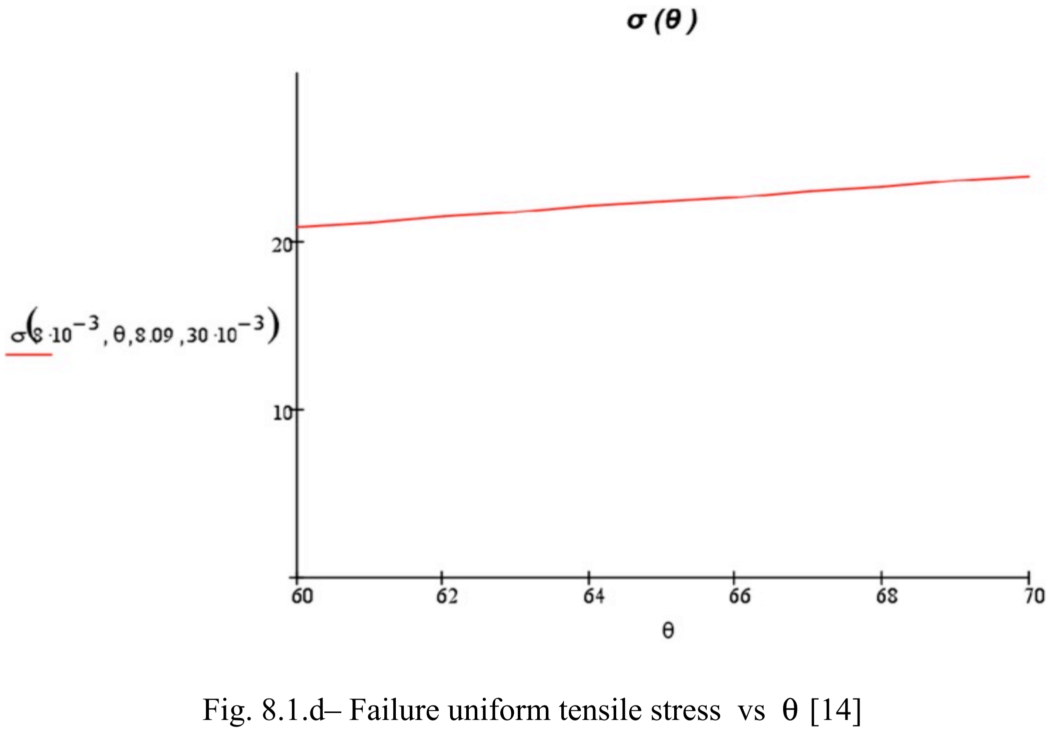

Fig. 8.1.d– Failure uniform tensile stress vs θ [14]

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